General Instructions:
(i) All questions are compulsory.
(ii) Figures to the right indicate full marks.
*(iii) The question paper consists of **9** questions into **FOUR** sections **A,B,C,D.***
- ***Section A** contains **2** questions of **1 mark** each.*
- ***Section B** contains **2** questions of **2 marks** each.*
- ***Section C** contains **3** questions of **4 marks** each.*
- ***Section D** contains **2** questions of **6 marks** each.*

SECTION A

Select and write the most appropriate answer from the given alternatives for each question:

1. Let N be the set of natural numbers and R be the relation in N defined as $R = \{(a,b): a = b-2, b > 6\}$, then **[1]**

(A) $(2,4) \in R$ (B) $(3,8) \in R$

(C) $(6,8) \in R$ (D) $(8,7) \in R$

Fill in the blanks in question:

2. A relation R in a set A is called ________________, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$ for all $a_1, a_2 \in A$. **[1]**

SECTION B

3. Write the smallest equivalence relation R on set A = {1, 2, 3}. **[2]**

4. Let * be a binary operation on N given by a * b = LCM (a, b) for all $a, b \in N$. Find 5 * 7. **[2]**

SECTION C

5. Let $A = \{1, 2, 3, \ldots\ldots, 9\}$ and R be the relation in $A \times A$ defined by $(a,b)R(c,d)$ if $a + d = b + c$ for $(a,b),(c,d)$ in $A \times A$. Prove that R is an equivalence relation. Also obtain the equivalence class $[(2,5)]$. **[4]**

6. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. **[4]**

7. Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a,b): a, b \in Z$ and $(a-b)$ is divisible by $5\}$. Prove that R is an equivalence relation. **[4]**

SECTION D

8. Let S be the set of all rational numbers except 1 and * be defined on S by a * b = a + b – ab $\forall a, b \in S$. Prove that :
(i) * is a binary on S.
(ii) * is commutative as well as associative. Also find the identity element of *. **[6]**

9. Show that the function $f : R \rightarrow \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$ is one-one and onto function. **[6]**

Answers

1 (C) $(6,8)\in R$	2 Symmetric
3 R = $\{(1, 1),(2, 2),(3, 3)\}$	4 LCM (5, 7) =35
5 A={1,2,3...,9} Consider (a,b) R (a,b) a+b=b+a Hence, R is reflexive. Consider (a,b) R (c,d) a+d=b+c =>c+b=d+a ⇒(c,d)R(a,b) Hence R is symmetric. Let (a,b) R (c,d) and (c,d) R (e,f) a+d=b+c and c+f=d+e ⇒a−c=b−d-- (1) c+f=d+e-- (2) Adding (1) and (2) a−c+c+f=b−d+d+e a+f=b+e (a,b)R(e,f) Hence R is transitive. Therefore, R is an equivalence relation. Now, p and q select from set A={1,2,3,....9} such that 2+q=5+p so q=p+3 Consider (1,4) (2,5) R (1,4)⇒2+4=5+1 [(2,5)=(1,4)(2,5),(3,6),(4,7),(5,8),(6,9)] is the equivalent class under relation R.	6 For one-one $f(x_1)=f(x_2)$ For onto: $y=\frac{4x+3}{6x-4}$ $f^{-1}(x)=\frac{4x+3}{6x-4}$ Function is inverse of itself
7 For Reflexive Relation (a,a)∈R. Since, (a−a)=0 is divisible by 5. Therefore, the relation is reflexive. For symmetric Relation (a,b)∈R⇒(b,a)∈R. (a,b)∈R⇒(a−b) is divisible by 5. Now, (b−a)=−(a−b) is also divisible by 5. Therefore, (b,a)∈R Hence, the relation is symmetric. For Transitive: If (a,b)∈R and (b,a)∈R⇒(a,c)∈R. (a,b)∈R⇒(a−b) is divisible by 5. (b,c)∈R⇒(b−c) is divisible by 5. then (a−c)=(a−b+b−c)=(a−b)+(b−c) is also divisible by 5. Therefore, (a,c)∈R. Hence, the relation is transitive. Therefore, the relation is equivalence relation.	8 (i) $S\in Q-\{1\}$. $a * b = a + b - ab \; \forall a,b\in S$. Sum, difference and product of rational numbers is a unique rational number. For each $(a, b)\in S\times S$, there exists a unique image $(a+b-ab)$ in S. * is a binary operation on S. (ii) Commutative $a * b = a + b - ab = b + a - ba = b*a$

	Associative $a*(b*c)=a*(b+c-bc)$ $=a+b+c-bc-ab-ac+abc$ and $(a*b)*c=(a+b-ab)*c$ $=a+b+c-bc-ab-ac+abc$ Let x be the identity element $a*x=x*a$ $a+x-ax=x+a-xa$ $x=0$ is the identity element of $*$
9 $f(x)=\frac{x}{1+\|x\|}$ **One – One** $f(x_1)=f(x_2)$ $\frac{x_1}{1+\|x_1\|}=\frac{x_2}{1+\|x_2\|}$ *Case (i)* If $x_1>0, x_2>0$ $\frac{x_1}{1+x_1}=\frac{x_2}{1+x_2}$ $x_1=x_2$ *Case (ii)* If $x_1<0, x_2<0$ $\frac{x_1}{1-x_1}=\frac{x_2}{1-x_2}$ $x_1=x_2$ *Case (iii)* If $x_1>0, x_2<0$ or $x_1<0, x_2>0$ $\frac{x_1}{1+x_1}=\frac{x_2}{1-x_2}$ $x_1-x_2x_1=x_2+x_2x_1$ $(x_1-x_2)=2x_1x_2$ This is not possible as $(x_1+x_2)>0$ and $(2x_1x_2)<0$, this is not valid equation. Hence, function is one-one in nature.	9 continued… $y=\frac{x}{1+\|x\|}=\begin{cases}\frac{x}{1+x} & ; x\ge 0\\ \frac{x}{1-x} & ; x<0\end{cases}$ So, $y=\frac{x}{1\pm x}$ $\Rightarrow x=\frac{y}{1\pm y}$ The domain of this is same as the range of $f(x)$ Therefore, it is onto.